The Math/Science Center

Factoring Trinomials (Quadratics):

Given the equation $a\,x^2 + b\,x + c = 0$, one way to find the solutions (i.e., values of X that make this equation true) is to factor the equation.

If the equation factors, then we will get the expression $(Px \pm M)\,(Qx \pm N) = 0$, with solutions $x = -{}^{M}/_{P}$ (or $+{}^{M}/_{P}$), $x = -{}^{N}/_{Q}$ (or $+{}^{N}/_{Q}$).

To see if an equation factors, we have to try different solutions to see what works.

<u>Case 1</u>: a = 1, c is positive (> 0)

In this case, if b is positive (>0), then the expression is $(x + M)\,(x + N)$ and

if b is negative (<0), then the expression is $(x - M)\,(x - N)$.

Now list the factors of b. Find the two factors whose sum is b: i.e., M + N = b

Example: $x^2 + 5\,x + 6 = 0$ Here a = 1, b = 5 > 0, c = 6 > 0

The factors of 6 are 1 • 6 Notice that 2 + 3 = 5 .
2 • 3 ←

The equation factors to $(x + 2)\,(x + 3) = 0$ and

the solutions are x = – 2, x = – 3.

<u>Case 2</u>: a = 1, c is negative (< 0)

In this case, the expression is $(x + M)\,(x - N)$ and b is the difference between the two factors of b , i.e., C – N = b or N – M = b .

If b is positive (>0) then M > N . If b is negative, then N > M .

Example: $x^2 + 5\,x - 6 = 0$ Here A = 1, B = 5 > 0, C = -6 < 0

Notice that 6 – 1 = 5 .

The equation factors to $(x - 1)\,(x + 6) = 0$; solutions x = 1, x = – 6

Note: Always FOIL the expression to check that you factored the trinomial correctly.

Case 3: $a \neq 1$ Follow the rules for c and b .

You will need to list the factors of $a \bullet c$.

Next find the factors of $a \bullet c$ whose sum (or difference) = b .

Say the factors are M and N. Rewrite the equation, replacing $b\,x$ with $M\,x + N\,x$.
You should be able to look at the equation and factor again.

Let's look at an example to see how this works.

Example: $9\,x^2 + 58\,x + 24 = 0$ Here a = 9, b = 58 > 0, c = 24 > 0, $a \bullet c = 216$

factors of 216:			
	$1 \bullet 216$	$4 \bullet 54$	$9 \bullet 24$
	$2 \bullet 108$	$6 \bullet 36$	$12 \bullet 18$
	$3 \bullet 72$	$8 \bullet 27$	

Notice that 4 + 54 = 58 .

Rewrite the equation: $9\,x^2 + 54\,x + 4\,x + 24 = 0$

Notice that 54 is divisible by 9, and 24 is divisible by 4. So you can factor again.

So the equation becomes: $9\,x^2 + (9 \bullet 6)\,x + 4\,x + (4 \bullet 6) = 0$

Or: $9\,x \bullet x + 9\,x \bullet 6 + 4\,x + 4 \bullet 6 = 0$

$9\,x\ (x + 6) + 4\,(x + 6) = 0$

$(9\,x + 4)\,(x + 6) = 0$ and the solutions are $x = -\,{}^{4}/_{9}$, $x = -6$

Questions: Factor the following trinomials

a) $x^2 + 8\,x + 16$ b) $x^2 - 1$ (Hint: b=0) c) $x^2 - x - 6$ d) $x^2 + x - 20$

e) $3\,x^2 - 14\,x + 8$ f) $4\,x^2 + 12\,x + 5$ g) $5x^2 - 18\,x - 8$ h) $x^2 + 5x + 4$

Answers: a) $(X + 4)^2$ b) $(X + 1)(X - 1)$ c) $(X + 2)(X - 3)$ d) $(X + 5)(X - 4)$ e) $(3X - 2)(X - 4)$ f) $(2X + 1)(2X + 5)$
g) $(5X + 2)(X - 4)$ h) $(X + 1)(X + 4)$