Logarithm Basics:

For every logarithmic equation of the form $N = \log_b(X)$ there is an exactly equivalent

exponential equation: $X = b^N$

$\log_b(X)$ is "the logarithm to the base b of X" and is the number to which you raise b in order to get X

So we can think of logarithms as powers of the base, b .

"Taking the logarithm of" and "raising to a power" are inverse operations, i.e., each "undoes" the other:

$N = \log_b(b^N)$ and $X = b^{\log_b(X)}$

For every rule for exponents there is a corresponding rule for logarithms:

Exponents	Logarithms
$b^0 = 1$	$\log_b(1) = 0$
$b^1 = b$	$\log_b(b) = 1$
$b^{-1} = 1/b$	$\log_b(1/b) = -1$
$b^N * b^M = b^{N+M}$	$\log_b(X * Y) = \log_b(X) + \log_b(Y)$
$b^N / b^M = b^{N-M}$	$\log_b(X/Y) = \log_b(X) - \log_b(Y)$
$(b^N)^M = b^{N*M}$	$\log_b(X^N) = N * \log_b(X)$

By convention: $\log(X) = \log_{10}(X)$

$\ln(X) = \log_e(X)$

where $e \cong 2.718$

Rule for changing base (e. g., to base 10):

$$\log_b(X) = \frac{\log(X)}{\log(b)}$$

Logarithms of integers up to 10:

N	2	3	4	5	6	7	8	9
log(N)	0.30	0.48	0.60	0.70	0.78	0.85	0.90	0.95

Exercises:

For each of 1 – 8, match the expression or equation with an equivalent expression or equation in a) – h).

1. $\log_5 25$ 2. $2^5 = X$ 3. $\log_5 5$ 4. $\log_2 1$

5. $\log_5 5^X$ 6. $\log_X 27 = 5$ 7. $8 = 2^X$ 8. $X^{-2} = 5$

a) 1 b) X c) $X^5 = 27$ d) $\log_2 X = 5$ e) $\log_2 8 = X$ f) $\log_X 5 = -2$

g) 2 h) 0

Simplify:

9. $\log_{10} 1000$ 10. $\log_2 16$ 11. $\log_8 1$ 12. $\log_9 9^5$ 13. $\log_{10} 0.01$

14. $\log_2 8$ 15. $\log_2 8$ 16. $\log_4 \frac{1}{4}$ 17. $\log_7 \frac{1}{49}$ 18. $\log_5 125$

19. $\log_9 3$ 20. $\log_9 27$ 21. $\log_{27} 9$ 22. $\log_{16} 64$ 23. $6^{\log_6 13}$

24. $\log_{1/4} {}^{1}/_{64}$ 25. $\log_{81} 3 * \log_3 81$ 26. $\log_{10}(\log_4(\log_3 81))$

Solve:

27. $|\log_3 X| = 2$ 28. $\log_4(3X - 2) = 2$ 29. $\log_8 (2X + 1) = -1$

30. $\log_{10}(X^2 + 21X) = 2$

Answers: 1. g 2. d 3. a 4. h 5. b 6. c 7. e 8. f 9. 3 10. 4
11. 0 12. 5 13. −2 14. 3 15. 3 16. −1 17. −2 18. 3 19. $^1/_2$ 20. $^3/_2$
21. $^2/_3$ 22. $^3/_2$ 23. 13 24. 3 25. 1 26. 0 27. $^1/_9$, 9 28. 6 29. $-^7/_{16}$ 30. −25, 4