Matrix Basics:

A matrix is an array of numbers.

The dimension or order of a matrix is given by **m x n**, where **m** is the number of rows and **n** is the number of columns

The elements of a matrix A are labeled with the row number and then the column number.

So $\mathbf{A}_{ij}$ is the element in row **i** and column **j**.

A matrix that has the same number of rows as columns is called a square matrix.

This matrix **A** has order 3 x 4

$$m = 3 \quad \begin{bmatrix} 5.4 & 6.2 & 8.1 & -4.7 \\ 11.9 & 15.7 & -1.6 & -0.5 \\ 0.2 & 1.1 & 0.8 & 16.4 \end{bmatrix} \quad n = 4$$

Here $\mathbf{A}_{23}$ is the number -1.6,
A_{32} is the number 1.1.

Question: Why do we use matrices anyway?

Answer: Matrices are useful when

1. Summarizing a data set (or sets) and relationships between data sets, and
2. Solving systems of linear equations.

In this handout, we will concentrate on basics of matrices.
Solving systems of linear equations will be the subject of another handout.

More Definitions:

A vector is a matrix that has one row or one column:

Row vector $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$ Column vector

To get the transpose of a matrix, interchange its rows and columns:

i.e., if $\mathbf{A} = \begin{bmatrix} 1 & -2 & 0 & 6 & 5 \\ -8 & 16 & 9 & -3 & 4 \\ 0 & 24 & 1 & 18 & 6 \end{bmatrix}$ then the transpose of **A**, $\mathbf{A}^T = \begin{bmatrix} 1 & -8 & 0 \\ -2 & 16 & 4 \\ 0 & 9 & 1 \\ 6 & -3 & 18 \\ 5 & 4 & 6 \end{bmatrix}$

A square matrix has the same number of rows as columns, e.g. $\begin{bmatrix} 1 & 8 \\ 6 & 3 \end{bmatrix}$

A diagonal matrix has nonzero elements only along the diagonal, e.g. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \\ 0 & 0 & 0 \end{bmatrix}$

Identity Matrices:

There is a special set of matrices, the set of **identity matrices.**

An identity matrix is a square matrix whose diagonal elements are all equal to 1.

These examples show the 2 x 2, 3 x 3, and 4 x 4 identity matrices. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

By convention, an identity matrix is written as **I**

Matrix Equality:

Two matrices are equal if (and only if) the matrices have the same numbers of rows and columns, and the corresponding elements within each matrix are all equal

Basic Matrix Operations:

Matrices can be added, subtracted, and multiplied by numbers.

Matrices are added element by element: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}$

and subtracted element by element: $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Only matrices having the **same order** can be added or subtracted.

To multiply a matrix by a number, multiply every element by that number. $2 \times \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 4 & 8 \end{bmatrix}$