Simple Projectile Motion:

In simple projectile motion, the only force is

- Due to gravity
- Constant in magnitude ($g = 9.8\ m/s^2$)
- Vertical

We can separate the motion of an object into horizontal and vertical components:

Horizontal: Motion with constant velocity

$$a_x = 0$$

$$v_{xf} = v_{xi}$$

$$x_f = x_i + v_{xi} t$$

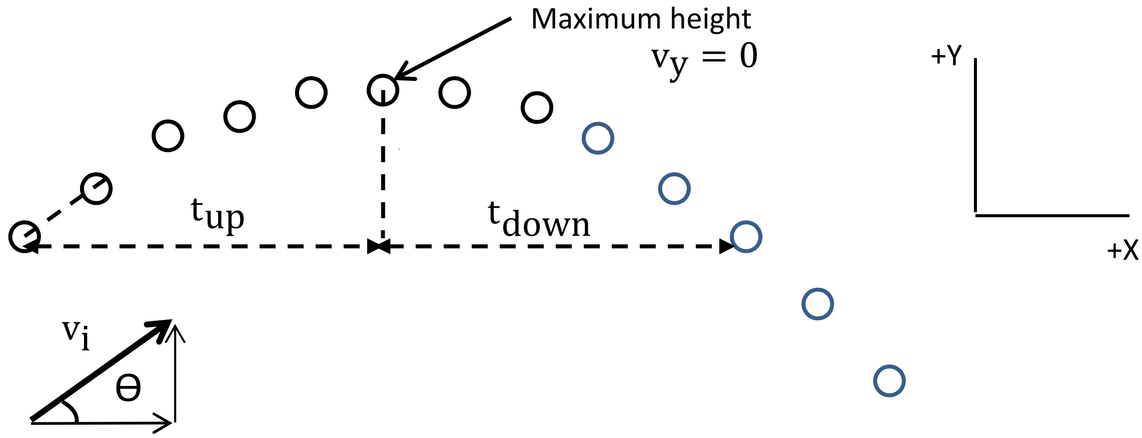
Vertical: Motion with constant acceleration

$$a_y = -g, \quad g = 9.8\, m/s^2$$

$$v_{yf} = v_{yi} - gt$$

$$y_f = y_i + v_{yi}\, t - \frac{1}{2} g\, t^2$$

where x_i (y_i) and x_f (y_f) are initial and final positions, v_i and v_f are initial and final speeds, and t is time ($t_i = 0$).



In the figure, a projectile is launched with initial speed v_i at angle Θ. So the initial vertical speed $v_{yi} = v_i \sin\Theta$

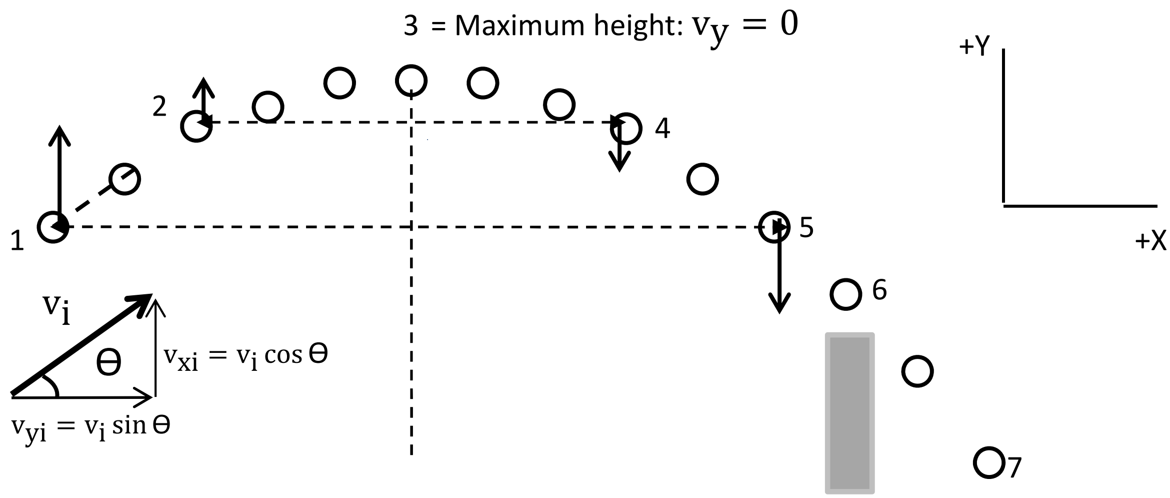
and the initial horizontal speed $v_{xi} = v_i \cos\Theta$

Tips:

Once launched, the projectile follows a parabolic path.

At the maximum height, the vertical speed will be zero (otherwise the projectile would go higher).

It takes the same time to reach the peak from any height (t_{up}) as it does to fall from the peak to that same height (t_{down}).



Tips (continued):

Because parabolas are symmetric, when the projectile is at the same heights, the speed going up will be the same as the speed going down.

The figure shows that the vertical speeds at positions 1 and 5 are the same, as are the vertical speeds for positions 2 and 4.

To find the maximum height, solve for the time when the vertical speed is zero. Then use that time to find the distance traveled.

It's often easier to break a problem into segments.

For example, the time required to go from position 1 in the figure to e.g., position 7 may be broken up into 2 calculations:

1. Calculate t_{up} , the time needed for the projectile to move from 1 to 3.
2. Calculate t_{down} , the time needed for the projectile to move from 3 to 7.

   t_{down} will be given by $y_i - y_f = \frac{1}{2} g\, t_{down}{}^2$
3. Add t_{up} + t_{down} to get the total time needed.

However, to see whether the projectile will clear an obstacle (e.g., figure position 6):

1. Calculate t_6 , the time needed to travel the horizontal distance (e.g., from 1 to 6).

   t_6 will be given by $x_f - x_i = v_{xi}\, t_6$
2. Then calculate the projectile height at t_6 ,and compare it to the height of the obstacle.

Remember that sometimes it's easiest to use the formula

$$v_{yf}{}^2 - v_{yi}{}^2 = 2\, g\, (y_i - y_f)$$